

Supergravity with Fayet-Iliopoulos terms and R-symmetry

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Abstract

The simplest examples of gauged supergravities are $N = 1$ or $N = 2$ theories with Fayet-Iliopoulos (FI) terms. FI terms in supergravity imply that the R-symmetry is gauged. Also the $U(1)$ or $SU(2)$ local symmetries of Kähler and quaternionic-Kähler manifolds contribute to R-symmetry gauge fields. This short review clarifies the relations.

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1 Introduction

In recent years, progress has been made towards building string configurations that describe theories close to the standard model, and models for early-universe cosmology. These involve configurations of intersecting branes with fluxes. The effective field theory descriptions lead to gauged supergravities. This means that the automorphism group of the supersymmetries is gauged by a physical spin-1 gauge field of the supergravity theory. The effective field theories for standard model building or cosmology often involve Fayet-Iliopoulos (FI) terms in $N = 1$ or $N = 2$, $D = 4$ supersymmetric models. In cosmological models, this allows to raise the value of the cosmological constant and as such obtain de Sitter vacua [1]. FI terms are the simplest examples of gauged supergravity theories, though the connection to gauged R-symmetry was not always emphasized, or was even neglected by some model builders. We recently [2] stressed this relation. This led to a better understanding of some theories for inflation and for the description of cosmic strings within supergravity [3]. Though the latter applications were discussed during the talk at this workshop, I will in these proceedings restrict myself to the explanation of R-symmetry and FI terms. I will include also $N = 2$ theories in this text, as the occurrence of R-symmetry for $N = 2$ is very similar to the situation for $N = 1$.

The automorphisms of the supersymmetries in various dimensions are determined by properties of the Clifford algebra. In 4 dimensions with N supersymmetries the group is $U(N)$, i.e. it is $U(1)$ for $N = 1$ and $SU(2) \times U(1)$ for $N = 2$. These $N = 2$ theories exist in very similar forms in 5 and 6 dimensions where the automorphism group is $USp(2) = SU(2)$. In 4 dimensions, the supersymmetry parameters can be split in 2 chiral parts $\epsilon = \epsilon_L + \epsilon_R$, which are essentially each others complex conjugates (strictly speaking: charge conjugates). The $U(1)$ R-symmetry is a phase transformation $\epsilon \rightarrow \exp(i\alpha\gamma_5)\epsilon$ or $\epsilon_L \rightarrow \exp(i\alpha)\epsilon_L$.

In section 2, a recapitulation of the basic ingredients of matter-coupled $N = 1$ supergravity is given [4]. It is indicated which data determine the theory completely, and where the FI term fits in this description. A short summary is also given for $N = 2$ supergravity. The superconformal formulation clarifies the structure of many equations, and in particular of the connection of R-symmetry with the FI term and other gauged symmetries. This is explained in section 3. Finally, in section 4 the relation of R-symmetry with symmetries in the geometry of the scalar manifolds is clarified. This involves the connections of Kähler and quaternionic-Kähler manifolds.

2 Matter-coupled $N = 1$ and $N = 2$ supergravity

Pure $N = 1$ supergravity contains a spin 2 and a spin 3/2 field: the graviton and the gravitino. One can add a number of vector multiplets, each having a spin 1 and a spin 1/2 field. The spin 1 fields that are introduced in this way are gauge fields for an arbitrary ordinary gauge group. Chiral multiplets contain a complex spin 0 field and a spin 1/2 field. These chiral multiplets can transform in a representation of the gauge group. I will restrict myself here to these multiplets, though recently it was emphasized [5] that tensor (linear) multiplets are a natural description for some string compactifications. A description of the couplings of tensor multiplets is given in [6]. It was shown already in [7] that the tensor multiplet couplings in $N = 1$ supergravity can be dualized to couplings of chiral multiplets. It is thus sufficient

to restrict to chiral, vector and the pure supergravity multiplets to have a complete description of $N = 1$ matter-coupled supergravities, though of course the tensor multiplet description may be more useful for some applications. Let me still emphasize that the discussion is restricted here to actions with at most terms with two spacetime derivatives.

The supersymmetry transformations of the fermions of supergravity theories contain a lot of information. The graviton is described by the vierbein e_μ^a , and its fermionic partner is the gravitino ψ_μ . The bosonic part of the supersymmetry transformation of the latter is

$$\delta\psi_{\mu L} = \left[\partial_\mu + \frac{1}{4}\omega_\mu^{ab}(e)\gamma_{ab} + \frac{1}{2}iA_\mu^B \right] \epsilon_L + \frac{1}{2}\kappa^2\gamma_\mu F_0 \epsilon_R, \quad (2.1)$$

where $M_P = \kappa^{-1}$ is the Planck mass. There are 3 objects that appear in this transformation law. The first one is the spin-connection ω_μ^{ab} , which depends on the graviton e_μ^a . The second one is A_μ^B , a composite gauge field for the U(1) R-symmetry as will be explained below. The third one is an auxiliary scalar F_0 . Both A_μ^B and F_0 depend on fields of other multiplets.

The vector multiplets contain the gauge vector fields W_μ^α , with gauge field strengths $F_{\mu\nu}^\alpha$, and the ‘gaugini’ λ^α . The chiral multiplets have complex scalars ϕ_i (we denote the complex conjugates as ϕ^i) and the matter fermions, whose left-chiral parts are denoted as χ_i . The fermions transform under supersymmetry as (again only bosonic terms)

$$\delta\lambda^\alpha = \frac{1}{4}\gamma^{\mu\nu}F_{\mu\nu}^\alpha\epsilon + \frac{1}{2}i\gamma_5 D^\alpha\epsilon, \quad \delta\chi_i = \frac{1}{2}\hat{\partial}\phi_i\epsilon_R - \frac{1}{2}F_i\epsilon_L. \quad (2.2)$$

Here appear auxiliary scalars D^α and F_i , which are functions of the physical fields.

Considering the bosonic part of the action

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{bos}} = & -\frac{1}{2}\kappa^{-2}R - \frac{1}{4}(\text{Re } f_{\alpha\beta})F_{\mu\nu}^\alpha F^{\mu\nu\beta} + \frac{1}{4}i(\text{Im } f_{\alpha\beta})F_{\mu\nu}^\alpha \tilde{F}^{\mu\nu\beta} \\ & -g_i{}^j(\hat{\partial}_\mu\phi^i)(\hat{\partial}^\mu\phi_j) - V(\phi, \phi^*), \end{aligned} \quad (2.3)$$

one can enumerate the objects that completely determine the theory:

1. the gauge group with generators enumerated by the values of the index α . The kinetic energy of the gauge fields is determined by holomorphic functions $f_{\alpha\beta}(\phi)$.
2. a representation of this gauge group enumerated by the values of the index i . The kinetic energy of the scalars is determined by a real Kähler potential $K(\phi, \phi^*)$, up to an equivalence:

$$g_i{}^j = g^j{}_i = \partial_i\partial^j K(\phi, \phi^*), \quad K(\phi, \phi^*) \approx K'(\phi, \phi^*) = K(\phi, \phi^*) + f(\phi) + f^*(\phi^*). \quad (2.4)$$

3. a holomorphic superpotential $W(\phi)$, which will determine the potential $V(\phi, \phi^*)$, see below.
4. for any U(1) factor in the gauge group: a constant ξ_α (the ‘FI constant’).

It is a general result of supergravity that the potential is given by the square of the supersymmetry transformations of the fermions, where the kinetic matrix determines how this square is formed. For the case of $N = 1$, this leads to

$$V = -3\kappa^2 F_0 F^0 + F_i g^i{}_j F^j + \frac{1}{2}D^\alpha(\text{Re } f_{\alpha\beta})D^\beta, \quad D^\alpha = (\text{Re } f)^{-1\alpha\beta}\mathcal{P}_\beta. \quad (2.5)$$

One recognizes here the auxiliary scalar expressions that appear in the fermion transformations (2.1), (2.2). The first two terms together are called the ‘ F -term’, while the last one is called the ‘ D -term’. The F -term depends on the superpotential W , and on the Kähler potential K (order κ^2 corrections). We do not need its explicit expression here. The D -term, on the other hand, depends on the gauge transformations and on K . Moreover, this contains the FI term, depending on the above-mentioned FI constants. The quantity \mathcal{P}_α is the ‘moment map’ of the symmetry indicated by the index α . It has a geometric meaning that we will discuss further in section 4. The value of this moment map involves the FI constants. However,

the addition of FI term gives restrictions in supergravity (e.g. on the superpotential). These have been neglected in previous papers on D-term cosmology [8]. One of the aims of [2] is to put this straight! The restriction on the superpotential will be explained in section 3.

That the D -term is related to R-symmetry is already clear by the fact that the moment map appears also in the expression of the composite $U(1)$ gauge field that we encountered in (2.1):

$$A_\mu^B = \omega^i \partial_\mu \phi_i + \omega_i \partial_\mu \phi^i + \kappa^2 W_\mu^\alpha \mathcal{P}_\alpha, \quad \omega^i \equiv -\frac{1}{2} i \kappa^2 \partial^i K, \quad \omega_i \equiv \frac{1}{2} i \kappa^2 \partial_i K. \quad (2.6)$$

Denoting the gauge transformations of the scalars as

$$\delta_G \phi_i = \Lambda^\alpha k_{\alpha i}(\phi), \quad \delta_G \phi^i = \Lambda^\alpha k_\alpha^i(\phi^*), \quad (2.7)$$

the value of the moment map can be written as

$$\mathcal{P}_\alpha = \frac{1}{2} i k_{\alpha i} \partial^i K - \frac{1}{2} i k_\alpha^i \partial_i K - \frac{3}{2} i \tilde{r}_\alpha(\phi) + \frac{3}{2} i \tilde{r}_\alpha^*(\phi^*). \quad (2.8)$$

These gauge symmetries do not necessarily leave the Kähler potential invariant, but due to the equivalence shown in (2.4), it can transform as the real part of a holomorphic function. That determines the value of the quantities $r_\alpha(\phi)$ in (2.8):

$$\delta_G K \equiv \Lambda^\alpha (k_{\alpha i} \partial^i K + k_\alpha^i \partial_i K) = 3 \Lambda^\alpha ([\tilde{r}_\alpha(\phi) + \tilde{r}_\alpha^*(\phi^*)]). \quad (2.9)$$

However, the above formula does not determine $r_\alpha(\phi)$ completely as only its real part occurs. As these functions should be holomorphic, the only remaining freedom are imaginary constants:

$$\tilde{r}_\alpha = \dots + \frac{1}{3} i \xi_\alpha. \quad (2.10)$$

To be consistent with the gauge group, this addition is only possible for Abelian factors, as can be understood from the relations in the next section, see (3.5).

Observe the 3 different types of terms contributing to the R-symmetry gauge field. First, there are the field-dependent shifts of bosons. The first terms of (2.6) are the pull-back of the Kähler connection, which will be further explained in section 4. The first terms in (2.8) are the covariantizations of these terms. Secondly, there are the possible contributions from a non-invariant Kähler potential, and finally there is the possibility of the constant FI terms. In string theory there are no free constants. E.g. coupling constants appear as vacuum expectation values of moduli fields. A possible way in which FI terms may appear is that one considers an effective theory in which some chiral multiplets are already integrated out. In the full theory with these chiral multiplets, the moment map may be non-zero and field-dependent. After fixing the fields that are integrated out, the resulting value of the moment map may be a constant that remains in the effective theory as a FI constant. This has been illustrated in more detail in [2].

Finally, let us consider the analogous facts for $N = 2$. The facts that we mention here are as well applicable to $D = 4$ [9–11], $D = 5$ [12, 13] or $D = 6$ [14]. We consider theories with n_V vector multiplets and n_H hypermultiplets. The former contain gauge fields A_μ^I for a gauge group, and there is one extra gauge field, the ‘graviphoton’, part of the pure supergravity multiplet, such that $I = 0, 1, \dots, n_V$. In 4 and 5 dimensions there are further complex or real scalar fields, which are not important for the R-symmetry story. The hypermultiplets contain each 4 real scalars that combine in quaternions. The scalar manifold is a quaternionic-Kähler manifold (some aspects of these manifolds are reviewed in section 4). We use here a basis of real scalars q^X , with $X = 1, \dots, 4n_H$. These may transform in a representation of the gauge group defined by the vector multiplets. The supersymmetry transformation of the gravitini contains a composite gauge field for the $SU(2)$ group in the automorphism group of the supersymmetries. Its value is very similar to (2.6):

$$\vec{\mathcal{V}}_\mu = \partial_\mu q^X \vec{\omega}_X + \kappa^2 A_\mu^I \vec{P}_I, \quad (2.11)$$

where $\vec{\omega}_X$ is a connection on the quaternionic-Kähler manifold determined by the scalars of the hypermultiplets. On the other hand, we have here a triplet moment map \vec{P}_I , related to the fact that there are 3 complex structures \vec{J}_X^Y on the quaternionic-Kähler manifold, while there is only 1 on the Kähler manifold, see section 4. Using the notation $\delta q^X = -\Lambda^I k_I^X$ (the minus sign is chosen for consistency with [12, 13]), one finds for these moment maps:

$$4n_H \kappa^2 \vec{P}_I = \vec{J}_X^Y D_Y k_I^X. \quad (2.12)$$

The value is undetermined when there are no hypermultiplets ($n_H = 0$) or in rigid supersymmetry ($\kappa = 0$). In these cases \vec{P}_I can be constants and the gauge symmetry implies that the corresponding gauge group should be U(1) or SU(2).

3 The superconformal origin of R-symmetry

The structure of matter-coupled supergravities is clarified by using a superconformal approach. This was developed for $N = 1$ in [15, 16], and a recent convenient summary is in [17]. For $N = 2$ see [10, 13, 14]. One first considers actions invariant under the superconformal group. This contains, apart from the super-Poincaré group, also dilatations and special conformal transformations, special supersymmetry and a local R-symmetry group. After gauge fixing of all these extra symmetries, the remaining theory is an ordinary super-Poincaré theory. Concerning the fields, one starts from the so-called Weyl multiplet. This contains gauge fields for all the superconformal symmetries. Some of these gauge fields will not be independent fields, but functions of the physical fields. E.g. in this multiplet will be a gauge field for the U(1) transformation that acts on the gravitino, $\delta_{U(1)} \psi_\mu = -\frac{1}{2} i \gamma_5 \Lambda_{U(1)} \psi_\mu$.

Apart from the physical multiplets, mentioned in the previous section, there is also an extra chiral multiplet that contains non-physical fields whose values are gauge-fixed in order to break the superconformal group at the end to the super-Poincaré group. The corresponding scalar is called the ‘conformon’ Y . Fixing its value introduces the Planck mass in the theory that was first of all conformal invariant. The gauge condition for dilatations fixes the modulus, while the phase of Y is fixed by a gauge choice for the U(1) R-symmetry in the superconformal group:

$$\text{Dilatation gauge: } YY^* e^{-\kappa^2 K/3} = \kappa^{-2}, \quad \text{U(1)-gauge: } Y = Y^*. \quad (3.1)$$

In general, one thus starts with $n + 1$ chiral multiplets, whose rigid symmetries define a rigid Kähler manifold. The couplings in this larger scalar manifold are restricted by the presence of a conformal symmetry. After gauge fixing, the remaining Kähler manifold has complex dimension n . The notation for the auxiliary field F_0 and the inclusion of the first term in (2.5) in the ‘F-term’ potential finds here its place, as they are related to the transformation of the fermion of the compensating chiral multiplet.

Without vector multiplets, there are still the first two terms in (2.6). But there is no gauge invariance left. The U(1) of the superconformal group is broken by the gauge choice (3.1). This is called ‘ungauged supergravity’. ‘Gauged supergravity’ is the situation in which physical fields enter in the expression A_μ^B . Hence, it occurs when $\mathcal{P}_\alpha \neq 0$. This is the case when chiral multiplets transform non-trivially under the gauge group or when FI terms are added.

Therefore, we now consider the gauging of symmetries in the superconformal formulation. Apart from the scalars ϕ_i , whose transformations were given in (2.7), there is now also the conformon. Its transformation law is parametrized as

$$\delta_G Y = Y r_\alpha(\phi) \Lambda^\alpha, \quad r_\alpha = \kappa^2 \tilde{r}_\alpha. \quad (3.2)$$

The way in which Y appears in the right-hand side is imposed by the fact that the gauge group should commute with the dilatations. The notation r_α is chosen, because one can check that with the dilatation gauge condition (3.1), this transformation of Y corresponds to the transformation (2.9) for the Kähler potential.

From (2.10) one sees that the FI term appear when the conformon undergoes a phase transformation under the gauge symmetry.

The $U(1)$ gauge choice (3.1) is not invariant under gauge transformations. This implies that in the final theory the gauge transformations get an extra contribution from the superconformal $U(1)$. The preservation of this gauge condition gives

$$\Lambda_{U(1)} = \frac{3}{2}\Lambda^\alpha(r_\alpha^* - r_\alpha) = \Lambda^\alpha(\kappa^2\mathcal{P}_\alpha + \omega^i k_{\alpha i} + \omega_i k_\alpha^i). \quad (3.3)$$

This implies that the gravitino now transforms under all the gauge transformations for which $r_\alpha \neq 0$. One can compute the gauge transformations of the auxiliary field (2.6) and finds that the gauge transformations of the fields ϕ_i and W_μ^α induce

$$\delta_G A_\mu^B = \partial_\mu \Lambda_{U(1)} \quad (3.4)$$

To prove this equation one has to use the ‘equivariance condition’ that follows from the fact that (3.2) satisfies the gauge algebra:

$$k_\alpha^j g_j^i k_{\beta i} - k_\beta^j g_j^i k_{\alpha i} + i f_{\alpha\beta}{}^\gamma \mathcal{P}_\gamma = 0. \quad (3.5)$$

The dilatational invariance imposes that the superpotential in the superconformal theory should be of the form

$$\mathcal{W} = Y^3 \kappa^3 W(\phi). \quad (3.6)$$

This superpotential should be invariant under the gauge symmetries. In view of (3.2) this implies that $W(\phi)$ should transform homogeneously under the gauge group

$$\delta_G W \equiv (\partial^i W) \Lambda^\alpha k_{\alpha i} = -3\Lambda^\alpha r_\alpha(\phi) W(\phi). \quad (3.7)$$

This is important for the cosmological scenarios that go under the name of D -term inflation [8, 18]. Also the KKLT scenario [19] has D -terms in its effective supergravity description. E.g. (3.7) implies that when one introduces FI terms for a symmetry, the superpotential should have a non-vanishing homogeneous transformation under this symmetry.

In $N = 2$ supergravity one has two extra multiplets in the superconformal approach. First there is an extra vector multiplet that contains the graviphoton apart from the conformon and its fermionic partners. Moreover, there is a compensating hypermultiplet. To describe n_H physical hypermultiplets, one starts with a hyper-Kähler manifold of quaternionic dimension $n_H + 1$ with conformal symmetry. This leads to a super-Poincaré theory with a quaternionic-Kähler manifold of dimension n_H . The quaternionic and conformal structure in this case completely determines the moment maps. In the case $n_H = 0$, the compensating hypermultiplet may still transform under the $U(1)$ or $SU(2)$ gauge symmetry. This is the origin of the FI terms similar to the phase transformation of the conformon for $N = 1$. A difference, however, is that $N = 1$ FI terms are still possible with non-trivial chiral multiplets, while for $N = 2$ supergravity the conformal and quaternionic structures do not allow arbitrary FI terms when there are non-trivial hypermultiplets. Note that in rigid $N = 2$ supersymmetry a FI term may still be added, because the restriction is intimately related to the presence of the conformal symmetry that one needs to get a supergravity theory out of the couplings of the hyper-Kähler manifold. The $SU(2)$ part of the superconformal group is gauge-fixed by fixing phases of the quaternion of the compensating multiplet. Any tri-holomorphic isometry on the $(n_H + 1)$ -dimensional hyper-Kähler manifold will also act on these phases. The gauge fixing then implies again that in the Poincaré supergravity theory, the gauge symmetries get contributions from the $SU(2)$ R-symmetry [13] similar to (3.3):

$$\vec{\Lambda}_{SU(2)} = -g\vec{\omega}_X k_I^X \Lambda^I + g\Lambda^I \vec{P}_I. \quad (3.8)$$

The auxiliary gauge field for the $SU(2)$ symmetry that appears in the supersymmetry transformation of the gaugini, (2.11) transforms under the gauge symmetries as an $SU(2)$ gauge vector with parameter (3.8), again due to the ‘equivariance condition’

$$2\kappa^2 \vec{P}_I \times \vec{P}_J + \frac{1}{2} k_I^X \vec{J}_X^Y g_{YZ} k_J^Z - f_{IJ}^K \vec{P}_K = 0. \quad (3.9)$$

4 Kähler and quaternionic-Kähler connections related to R-symmetry

The R-symmetry is related to reparametrizations of geometric properties of the scalar manifold. In this final section, I give a recapitulation of the relevant facts. The supersymmetry algebra imposes that these manifolds have complex structures. For the chiral multiplets I adopted already a notation in which all the coordinates, say ϕ^x , are split in holomorphic coordinates $\phi^x = \{\phi_i, \phi^i\}$. This means that the complex structure is diagonalized:

$$J_x^y = \begin{pmatrix} J^i_j & 0 \\ 0 & J_i^j \end{pmatrix}, \quad J^i_j = -J_i^j = -i\delta_i^j, \quad J_x^y J_y^z = -\delta_x^z. \quad (4.1)$$

The scalars of the hypermultiplets in $N = 2$, denoted by q^X , form a manifold with a quaternionic structure, i.e. there is the triplet \vec{J}_X^Y such that for arbitrary 3-vectors $\vec{\alpha}$ and $\vec{\beta}$,

$$\vec{\alpha} \cdot \vec{J} \vec{\beta} \cdot \vec{J} = -\mathbb{1}_{4r} \vec{\alpha} \cdot \vec{\beta} + (\vec{\alpha} \times \vec{\beta}) \cdot \vec{J}. \quad (4.2)$$

Another Ward identity of supergravity implies that these complex structures are related to the curvature of the connection that appears in the gravitino transformation law, see e.g. for $N = 1$: (2.1), (2.6). The expressions are:

$$\begin{aligned} N = 1 & : R_i^j \equiv \partial_i \omega^j - \partial^j \omega_i = -\kappa^2 J_i^k g_k^j, & R^i_j \equiv \partial^i \omega_j - \partial_j \omega^i = -\kappa^2 J^i_k g^k_j, \\ N = 2 & : \vec{R}_{XY} \equiv 2\partial_{[X} \vec{\omega}_{Y]} + 2\vec{\omega}_X \times \vec{\omega}_Y = -\frac{1}{2} \kappa^2 \vec{J}_X^Z g_{ZY}. \end{aligned} \quad (4.3)$$

Gauged isometries have to preserve the complex structures. They are therefore holomorphic in $N = 1$, and for the hypermultiplets of $N = 2$ there is a similar requirement that they preserve the quaternionic structure. In each case it can be shown that this implies the existence of moment maps:

$$N = 1 : \partial_i \mathcal{P}_\alpha = J_i^j g_j^k k_{\alpha k}, \quad N = 2 : D_X \vec{P}_I \equiv \partial_X \vec{P}_I + 2\vec{\omega}_X \times \vec{P}_I = -\frac{1}{2} \vec{J}_{XY} k_I^Y. \quad (4.4)$$

These moment maps gives the amount by which a gauge symmetry contributes to the R-symmetry. We saw that the value of the auxiliary D -field is proportional to it, see (2.5) and also in $N = 2$ a triplet auxiliary field of vector multiplets is proportional to \vec{P}_I . For $N = 1$ this moment map is only determined up to a constant, and the latter is then the FI-term. For $N = 2$ the curvature of the quaternionic-Kähler manifold does not allow this, and FI terms are only possible without physical hypermultiplets in supergravity.

For $N = 1$ one uses the Kähler potential to describe the manifold. The latter is not uniquely defined, but can transform under ‘Kähler transformations’ as in (2.4). In the conformal approach these originate from a change of definition of the conformon from Y to $Y' = Y \exp[\kappa^2 f(\phi)/3]$. The transformation thus also changes the value of r_α . One can check easily that

$$\tilde{r}'_\alpha = \tilde{r}_\alpha + \frac{1}{3} (\partial^i f) k_{\alpha i}, \quad \omega'^i = \omega^i - \frac{1}{2} i \kappa^2 \partial^i f(\phi), \quad \mathcal{P}'_\alpha = \mathcal{P}_\alpha. \quad (4.5)$$

The quantity ω^i is thus the gauge field of these Kähler transformations. On the other hand, A_μ^B is as well the gauge field for the part of the gauge transformations that act as R-symmetry, (3.3), as for the pull-back of the Kähler transformations to spacetime, as

$$A_\mu'^B = A_\mu^B + \kappa^2 \partial_\mu \text{Im } f. \quad (4.6)$$

A similar situation holds for $N = 2$. In that case, one may rotate the $SU(2)$ vector quantities depending on a triplet of functions $\vec{l}(q)$, and $\vec{\omega}_X$ is the gauge field for these rotations:

$$\delta_l \vec{J}_X^Y = \vec{l} \times \vec{J}_X^Y, \quad \delta_l \vec{P}_I = \vec{l} \times \vec{P}_I, \quad \delta_l \vec{\omega}_X = -\frac{1}{2} \partial_X \vec{l} + \vec{l} \times \vec{\omega}_X. \quad (4.7)$$

The field \vec{V}_μ , (2.11), is, apart from gauge field of the $SU(2)$ part of the gauge transformations, (3.8), also the gauge field for the pull-back to spacetime of these l -reparametrizations:

$$\delta_l \vec{V}_\mu = \partial_\mu q^X \delta_l \vec{\omega}_X + g\kappa^2 A_\mu^I \delta_l \vec{P}_I = -\frac{1}{2} \partial_\mu \vec{l}(q(x)) + \vec{l} \times \vec{V}_\mu. \quad (4.8)$$

Note that when one does not gauge any isometry on the Kähler manifold ($N = 1$) or on the quaternionic-Kähler manifold ($N = 2$), then there is no $\Lambda_{U(1)}$ or $\Lambda_{SU(2)}$, but still the A_μ^B or \vec{V}_μ gauges the reparametrizations on the scalar manifolds.

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